

Review of “Chap.1 Origin of magnetism”

1.1 Magnetism of free atoms and ions

- Magnetism can be described by L , S and J of atoms or ions.
- Magnetization is determined by Pauli principle and Hund rule.

1.2 Localized electron magnetism in solids

- Magnetization for 3d-systems is mainly determined by S .
- While, for 4f-ions and metals magnetization is determined by J .
- Ferromagnetism is originated from the exchange interaction between atomic moments, and Curie temperature is proportional to the exchange integral.

1.3 Itinerant electron magnetism in metals

- Band theory is more applicable.
- For 4f electron metals and alloys, the indirect exchange interaction through conduction electrons is important.

2. Magnetic properties of ferromagnets

2.1 Magnetic anisotropy

2.2 Magnetoelastic effects

2.3 Magnetic domains and domain walls

2.4 Magnetization process

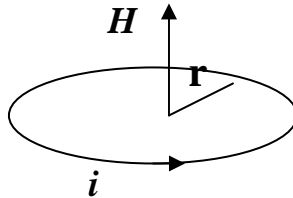
Unit system (MKSA)

1. Magnetic field (H)

Ampere Law

$$\oint H ds = i[A]$$

$$H = i / 2\pi r \quad [A/m]$$



2. Magnetic moment (μ)

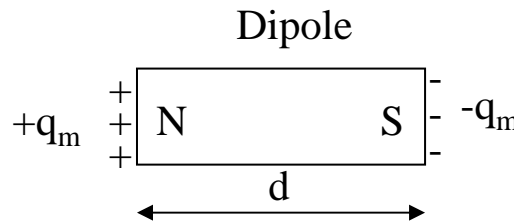
$$q_m \quad [Wb]$$

Dipole moment

$$\mu = q_m d \quad [Wb \cdot m]$$

On the other hand, $\mu = \mu_0 A i$, $\mu_0: [H/m]$ **Consistent**

Thus, $[(H/m) m^2 A] = [Wb \cdot m] \rightarrow [H] = [Wb/A]$



From electricity

$$\Phi = L \cdot i$$

Magnetic flux

[Wb]

Inductance

[H]

Current

[A]

3. Magnetization (I)

Magnetic moment per unit volume;

$$I = \mu / V = [Wb \cdot m / m^3] = [Wb/m^2] = [T]$$

4. Magnetic flux density (B)

$$B = \mu_0 H + I \quad [Wb/m^2]$$

$$= \mu_0 (H + M), \quad M [A/m]$$

5. Susceptibility ()

$$= I / H \quad [(Wb/m^2)/(A/m)]$$

$$= [Wb/A m] = [H/m]$$

6. Permeability (μ)

$$\mu = B / H \quad [(Wb/m^2)/(A/m)]$$

$$= [Wb/A m] = [H / m]$$

MKSA and CGS Units

Physical quantity	Symbol	MKSA unit	CGS unit	Conversion (MKSA/CGS)
Magnetic pole	q_m	Wb	emu	1.257×10^{-7}
Magnetic flux		Wb	Mx	10^{-8}
Magnetic moment	μ	Wb · m	G · cm ³	1.257×10^{-9}
Magnetization	I	T	G	1.257×10^{-3}
Magnetic flux density	B	T	G	10^{-4}
Magnetic field	H	A/m	Oe	79.6
Susceptibility		H/m	-	1.579×10^{-5}
Permeability	μ	H/m	-	μ_0

2.1 Magnetic anisotropy

Heisenberg exchange interaction is isotropic.

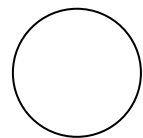
$$H = -\sum_{i<j} J_{ij} \mathbf{S}_i \mathbf{S}_j = -\sum_{i<j} J_{ij} (\mathbf{S}_{ix} \mathbf{S}_{jx} + \mathbf{S}_{iy} \mathbf{S}_{jy} + \mathbf{S}_{iz} \mathbf{S}_{jz})$$

The real magnetic materials, however, exhibit anisotropy, which is essentially caused by the **spin-orbit coupling**. Spin-orbit coupling means that the direction of spin depends on the electronic orbital shape and orientation..



While, the orbital states are influenced by the electronic configurations and the crystalline symmetries
Thus, magnetic anisotropy depends on the electronic orbital shape and the crystalline symmetry.

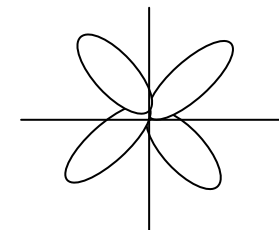
Orbital shapes



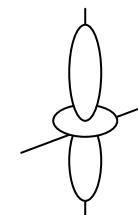
s-orbital
($l = 0$)



p-orbital
($l = 1$)



d_{ϵ}



$d_{\gamma} (d_{x^2-y^2})$

d-orbital
($l = 2$)

2.1.1 Observations

Magnetization curves for Fe, Ni and Co

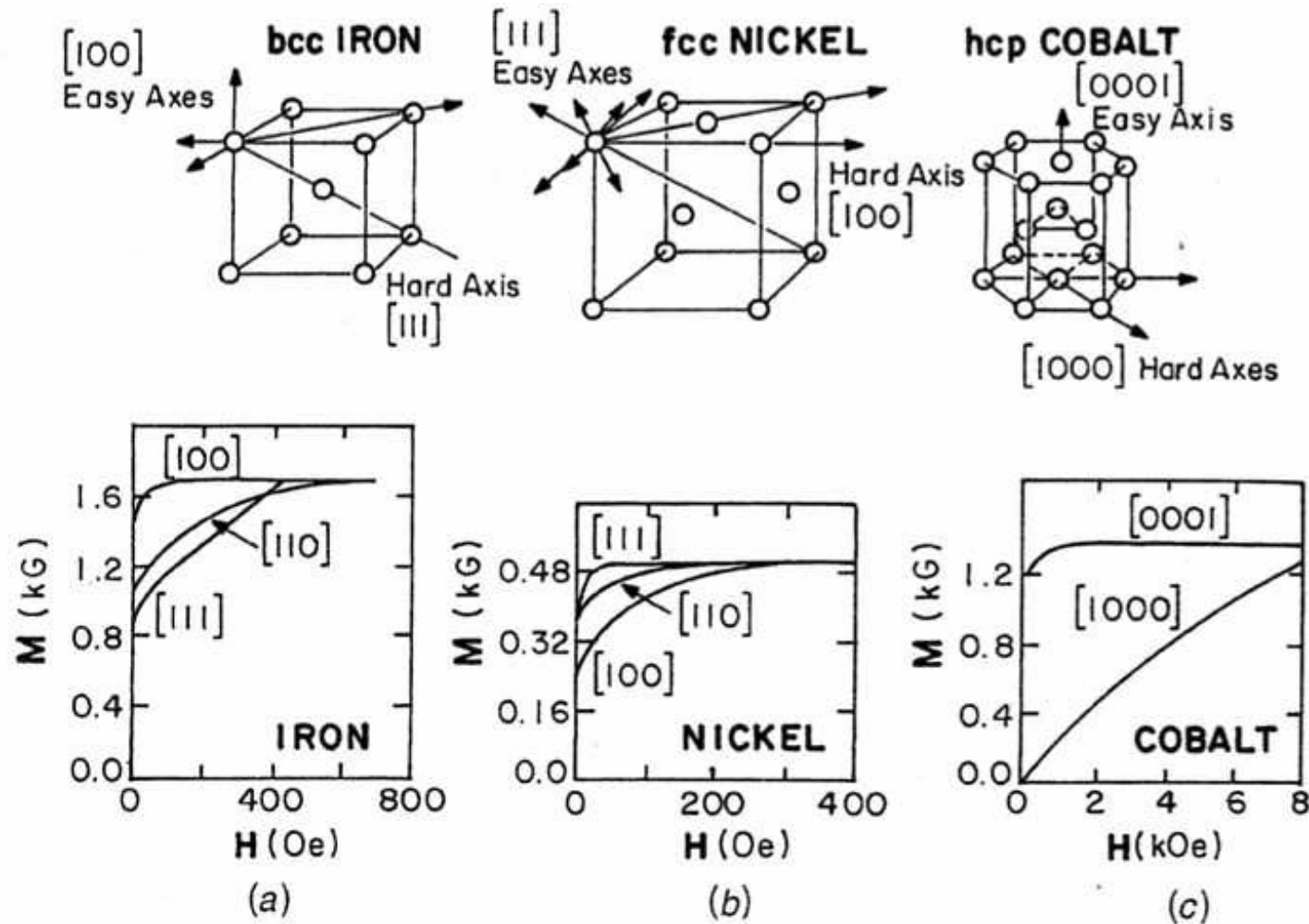


Figure 6.1 Crystal structure showing easy and hard magnetization directions for Fe(a), Ni(b), and Co(c), above. Respective magnetization curves, below.

In general uniaxial anisotropy is larger than cubic anisotropy.

TABLE 6.1 Magnetic Anisotropy Constants K_1 and K_2 for Selected Materials^a

	(T = 4.2 K)		(RT)		
	K ₁	K ₂	K ₁	K ₂	
3d Metals					
Fe	5.2 × 10 ⁵	-1.8 × 10 ⁵	4.8 × 10 ⁵	-1.0 × 10 ⁵	Cubic Hexagonal Cubic
Co ^u	7.0 × 10 ⁶	1.8 × 10 ⁶	4.1 × 10 ⁶	1.5 × 10 ⁶	
Ni	-12 × 10 ⁵	3.0 × 10 ⁵	-4.5 × 10 ⁴	-2.3 × 10 ⁴	
Ni ₈₀ Fe ₂₀	—	—	-3 × 10 ³	—	
Fe ₅₀ Co ₅₀			-1.5 × 10 ^{5b}		
4f Metals					
Gd ^u	-1.2 × 10 ⁶	+8.0 × 10 ⁵	+1.3 × 10 ⁵	—	Hexagonal (Uniaxial)
Tb ^u	-5.65 × 10 ⁸	-4.6 × 10 ⁷	—	—	
Dy ^u	-5.5 × 10 ⁸	-5.4 × 10 ⁷	—	—	
Er ^u	+1.2 × 10 ⁸	-3.9 × 10 ⁷	—	—	
Spinel Ferrites					
Fe ₃ O ₄	-2 × 10 ⁵	—	-0.9 × 10 ⁵	—	Cubic
NiFe ₂ O ₄	-1.2 × 10 ⁵	—	-0.7 × 10 ⁵	—	
MnFe ₂ O ₄	≈ -4 × 10 ⁵	≈ -3 × 10 ⁵	-3 × 10 ⁴	—	
CoFe ₂ O ₄	+ 10 ⁷	—	2.6 × 10 ⁶	—	
Garnets					
YIG	-2.5 × 10 ⁴	—	1 × 10 ⁴	—	Cubic
GdIG	-2.3 × 10 ⁵	—	—	—	
Hard Magnets					
BaO·6Fe ₂ O ₃ ^u	4.4 × 10 ⁶	—	3.2 × 10 ⁶	—	Uniaxial
Sm ^u Co ₅	7 × 10 ⁷	—	1.1 - 2.0 × 10 ⁸	—	
Nd ^u Co ₅	-4.0 × 10 ⁸	—	1.5 × 10 ⁸	—	
Fe ₁₄ Nd ₂ B ^u	-1.25 × 10 ^{8c}	—	5 × 10 ⁷	—	
Sm ₂ Co ₁₇ ^u	—	—	3.2 × 10 ⁷	—	
TbFe ₂	—	—	-7.6 × 10 ⁷	—	

2.1.2 Phenomenology of anisotropy

1) Uniaxial anisotropy

Co crystals show negligible anisotropy in the basal plane. So, Co is described as **uniaxial** with a preference for magnetization along c axis.

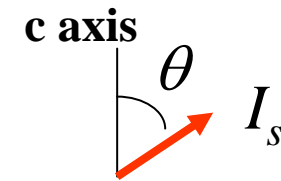
Uniaxial anisotropy energy per volume is expressed as a power series of the form

$$E_A = \sum_n K_{un} \sin^{2n} \theta$$

$$= K_{u0} + K_{u1} \sin^2 \theta + K_{u2} \sin^4 \theta + \dots \quad (2.1.1)$$

Usually, $K_{u1} > K_{u2}$

$K_{u1} > 0$ implies an easy axis.



	K_{u1} [J/m ³]	K_{u2} [J/m ³]
Co	4.1×10^5	1.5×10^5

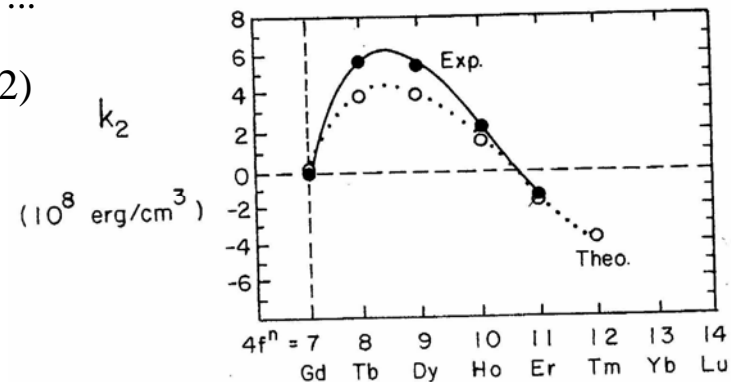
E_A can be also expressed using **Legendre polynomials**.

$$E_A = \sum_{l=0} k_l g_l(\alpha) = k_0 + k_2(\alpha^2 - 1/3) + k_4(\alpha^4 - 6/7 + 3/35) + \dots$$

where $g_l(\alpha) = \sum_{m=-l}^l A_l^m Y_l^m(\alpha)$ (2.1.2)

Y_l^m is spherical harmonics

$$k_2 = -K_{u1} - (8/7) K_{u2}, \quad k_4 = K_{u2} \quad (2.1.3)$$



Anisotropy field

We consider the **uniaxial materials**.

In order to describe quantitatively the hard-axis magnetization process in uniaxial materials, the Zeeman energy $-I_s H \sin \theta$ is added to the uniaxial anisotropy.

By using Eq. (2.1.1) for uniaxial anisotropy, the total energy is

$$E = E_A - I_s H \sin \theta = K_{u1} \sin^2 \theta + K_{u2} \sin^4 \theta - I_s H \sin \theta$$

$\partial E / \partial \theta = 0$ gives

$$2K_{u1} \sin \theta \cos \theta + 4K_{u2} \sin^3 \theta \cos \theta = I_s H \cos \theta$$

Magnetization component in the field direction is $m = I / I_s = \sin \theta$. Thus, we obtain

$$I_s H = 2K_{u1} m + 4K_{u2} m^3$$

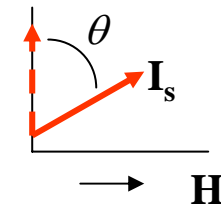
For saturation $m = 1$, which gives

$$I_s H = 2K_{u1} + 4K_{u2}$$

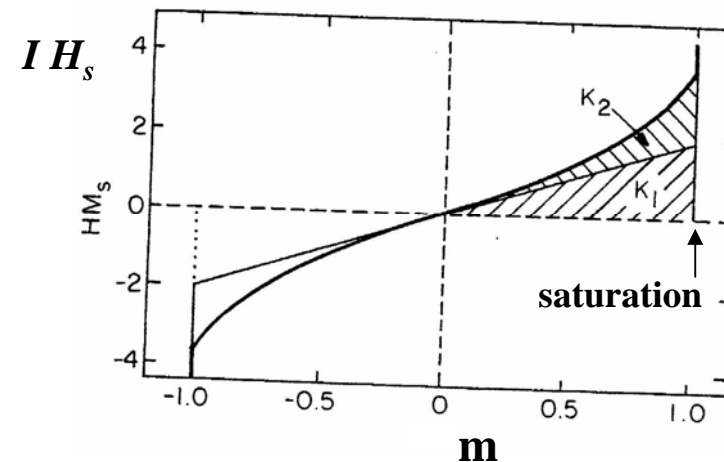
If K_{u2} is neglected, we obtain the anisotropy field H_a

$$H_a = 2K_{u1} / I_s \quad (2.1.4)$$

Easy axis



Hard axis

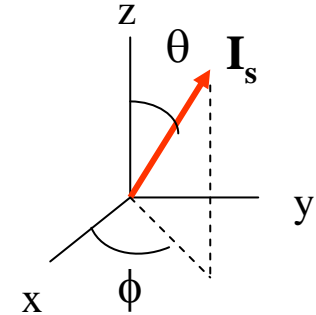


(2) Cubic anisotropy

$$E_A = K_0 + K_1 (\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2) + K_2 \alpha_1^2 \alpha_2^2 \alpha_3^2 + \dots$$

$\alpha_1, \alpha_2, \alpha_3$ are directional cosine.

$$\alpha_1 = \sin \theta \cos \phi, \quad \alpha_2 = \sin \theta \sin \phi, \quad \alpha_3 = \cos \theta, \quad \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$$



In general $K_1 > K_2$

Thus, [100] is easy axis for $K_1 > 0$: Fe case

[111] is easy axis for $K_1 < 0$: Ni case

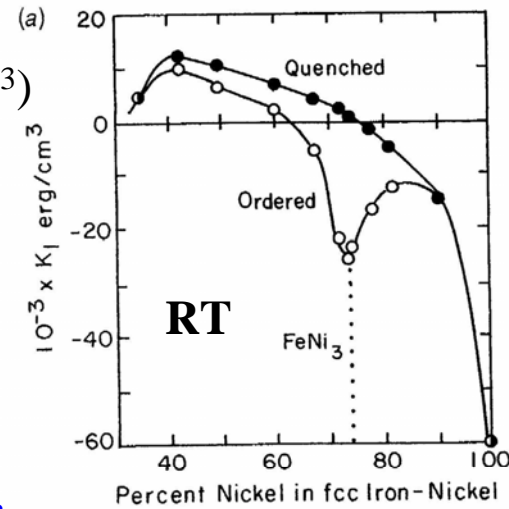
	K_1 [J/m ³]	K_2 [J/m ³]
Fe	4.8×10^4	$- 1.0 \times 10^4$
Ni	$- 4.5 \times 10^3$	$- 2.3 \times 10^3$
Ni ₈₁ Fe ₁₉	~ 0	~ 0

Cubic: [100], [010] and [001] are equivalent.

FCC Fe-Ni system

Anisotropy depends on
the order-disorder transformation.

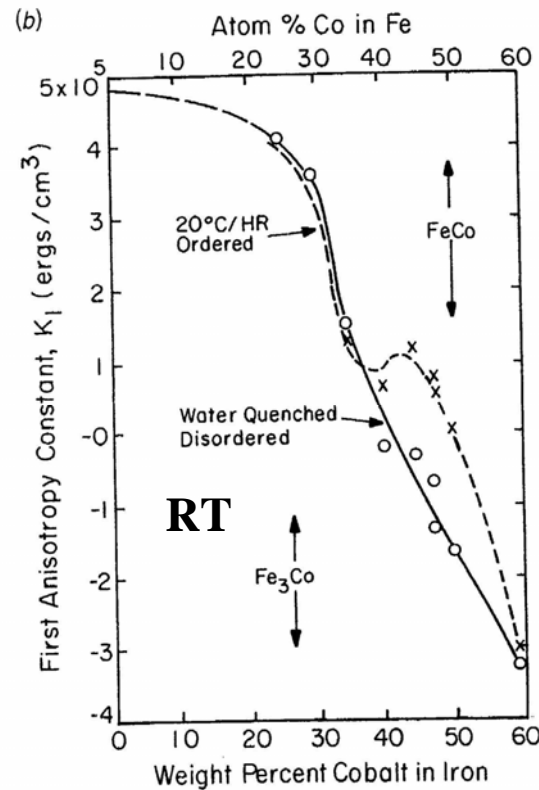
$10^{-3}K_1$ (erg/cm³)



→ Ni(%) in fcc Fe-Ni

BCC Fe-Co system

K_1 (erg/cm³)



→ Co(%) in Fe

Figure 6.9 First-order anisotropy constant for (a) FCC NiFe alloys and (b) BCC FeCo alloys (Hall 1959) at room temperature.

2.2 Physical origin of magnetocrystalline anisotropy

Magnetic anisotropy of metals is difficult to discuss theoretically. In insulators and 4f metals, however, where the electronic states important for magnetism are well defined functions with good quantum numbers due to **the localized magnetic electrons**, magnetocrystalline anisotropy can be discussed using **crystalline field**.

Crystalline field theory

Single ion anisotropy: Crystal field splitting

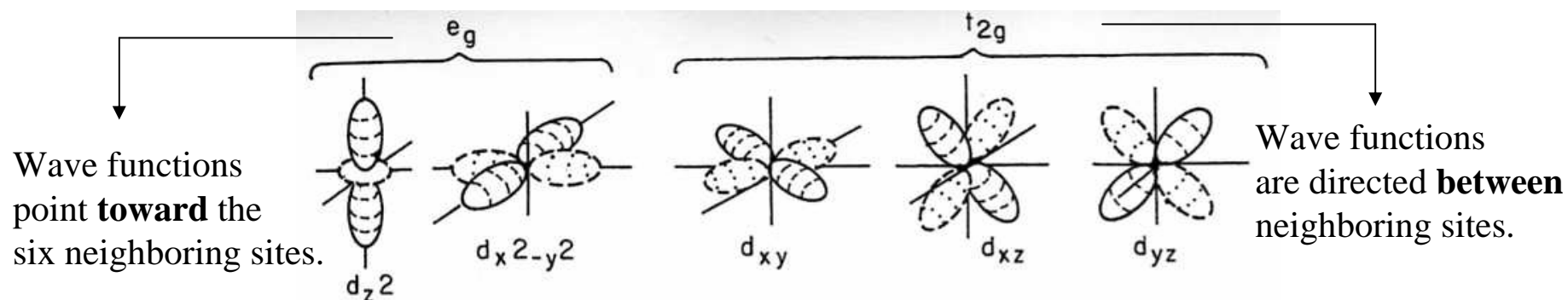
For transition metal ions, magnetic anisotropy is usually treated by examining the crystal field splitting of valence states of the magnetic ion of interest and **adding spin-orbit coupling as a perturbation**.

The 3d electrons have 5 orbitals, of which wave functions can be written in spherical coordinate as

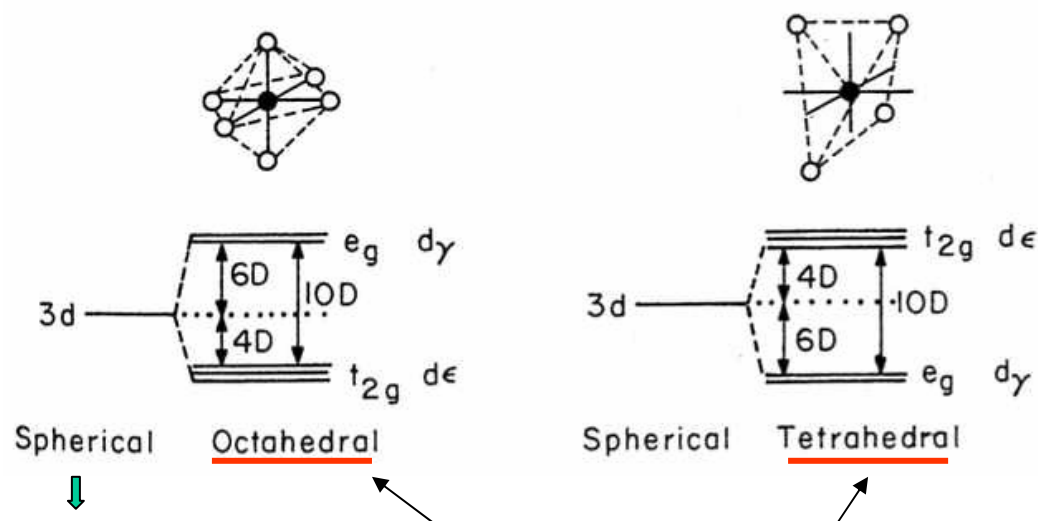
$$\begin{array}{lcl}
 d_{z^2} \propto \Psi_0 & & \\
 d_{x^2-y^2} \propto [\Psi_2 + \Psi_{-2}] & \left. \vphantom{\begin{array}{l} d_{z^2} \\ d_{x^2-y^2} \\ d_{yz} \\ d_{zx} \\ d_{xy} \end{array}} \right\} & d\gamma (e_g) \\
 d_{yz} \propto [\Psi_1 - \Psi_{-1}] & & \\
 d_{zx} \propto [\Psi_1 + \Psi_{-1}] & \left. \vphantom{\begin{array}{l} d_{yz} \\ d_{zx} \\ d_{xy} \end{array}} \right\} & d\varepsilon (t_{2g}) \\
 d_{xy} \propto [\Psi_2 - \Psi_{-2}] & &
 \end{array}
 \quad
 \begin{array}{l}
 \uparrow \qquad \uparrow \\
 \text{Bethe (Mulliken) Notation}
 \end{array}$$

where,
 $\Psi_0 \propto 3(\cos^2\theta - 1/3)$
 $\Psi_{\pm 1} \propto \sin\theta \cos\theta \exp(\pm i\phi)$
 $\Psi_{\pm 2} \propto \sin^2\theta \exp(\pm 2i\phi)$

Topologies of the d-orbital wave functions



$3d^4$



5 orbitals are degenerated.

Energy levels are splitted by the crystalline field.

Crystalline field splitting is larger than the spin-orbit coupling in 3d electrons.

Rare-earth (R) metals

For rare-earth (R) metals and R-intermetallic compounds, **the crystalline field splitting are weak** compared to spin-orbit energy. Therefore, **the magnetic anisotropy of R metals becomes large**. Crystalline field effects are calculated as perturbations on the atomic energies with 4f states:

$$\langle \Psi_J^* | \mathbf{H}_{cf} | \Psi_J \rangle \propto \alpha_l \langle r^l \rangle O_l^m(\mathbf{J}) \quad , \quad (2.1.5)$$

where, crystalline field Hamiltonian is

$$\mathbf{H}_{cf} = -e \sum_i \mathbf{V}(\mathbf{r}_i)$$

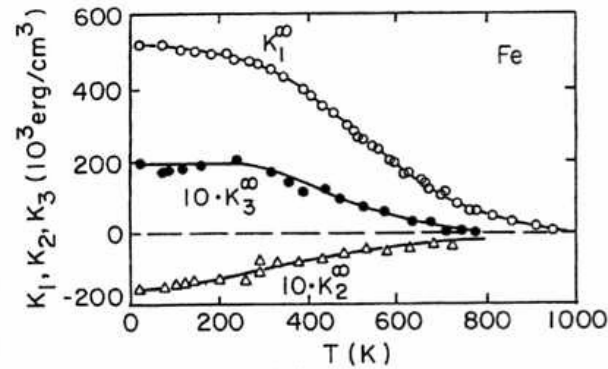
α_l : Stevens factor, the l -th moments of the wave functions

Operator O_l are related to the Legendre polynomials which describe the symmetry of the crystalline field, which are listed in the table of the some textbooks of magnetism.

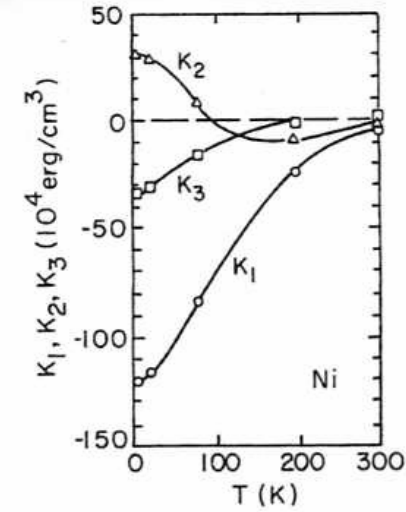
For example, in the case of $l = 2$,

$$\alpha_2 \langle r^2 \rangle O_2^0(\mathbf{J}) = B_2^0 O_2^0 = B_2^0 [3J_z^2 - J(J+1)]$$

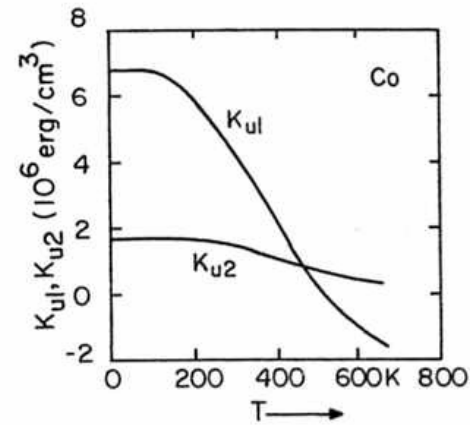
Temperature dependence of anisotropy



(a)



(b)



(c)

Temperature dependence of the anisotropy is larger than that of the magnetization.

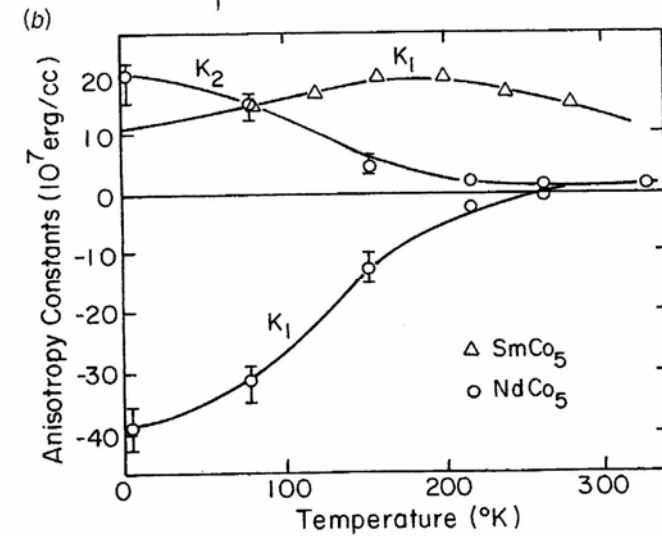
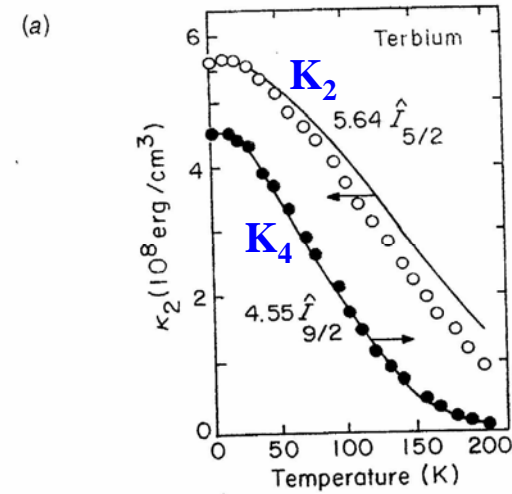
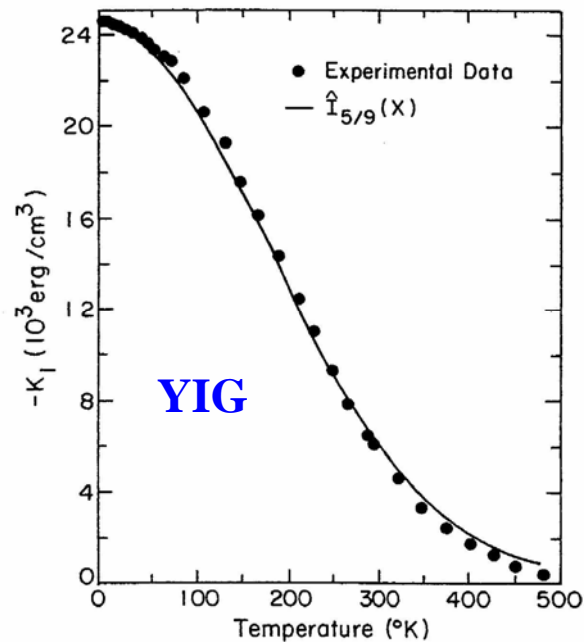
$$K_i(T)/K_i(0) = [I(T)/I_0]^{l(l+1)/2}$$

$l = 2$ for uniaxial anisotropy

$l = 4$ for cubic uniaxial anisotropy $K \propto I^{10}$

After Callen and Callen

$K_i(T)/K_i(0) = I_{l+1/2}(x)$: Bessel function



2.1.3 Measurement of the anisotropy

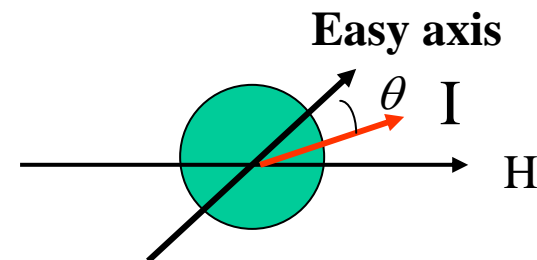
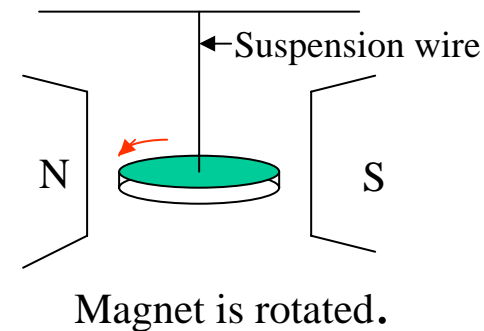
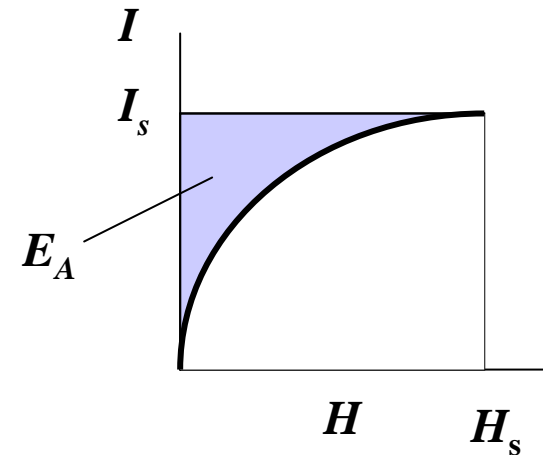
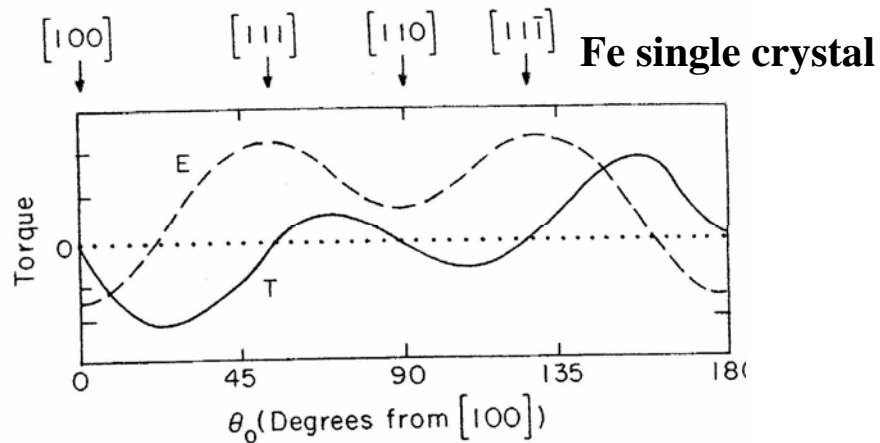
1) Magnetization curve

$$E_A = \int_0^{I_s} I(H) dI$$

2) Torque measurement for a single crystal

$$T = -\partial E_A / \partial \theta = -I H \sin \theta$$

E_A is the anisotropy energy.



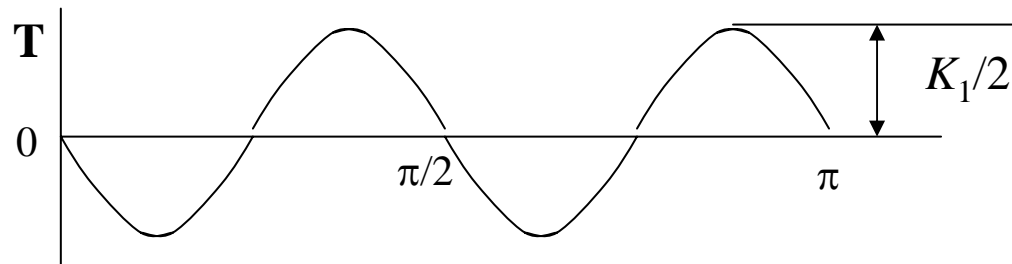
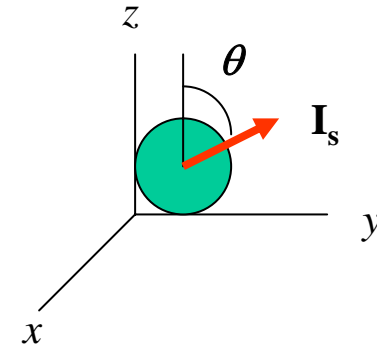
Cubic anisotropy

(100) plane

$$E_A = K_1(\sin^2 \theta - \sin^4 \theta)$$

$$= (K_1/4) \sin^2 2\theta$$

$$T = -\partial E_A / \partial \theta = -(K_1/2) \sin 4\theta$$



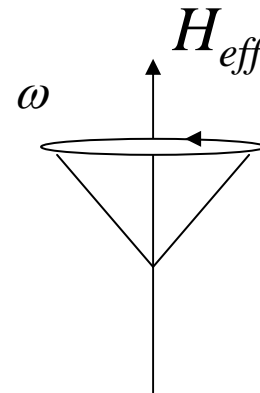
3) Ferromagnetic resonance

$$dI / dt = -\gamma I \times H_{eff} \quad H_{eff} = H + H_A$$

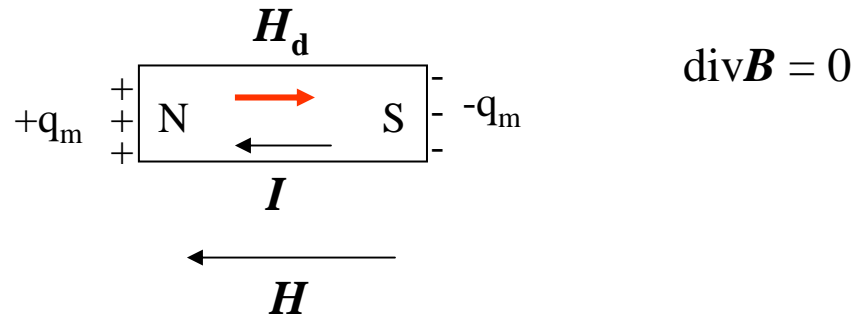
$$\omega_0 = \gamma H_{eff}$$

$$H_t = H_{eff} + h e^{i\omega t} \quad I = I_s + m e^{i\omega t}$$

$\omega = \omega_0$: resonance frequency



2.1.4 Shape anisotropy



Demagnetization field: \mathbf{H}_d

$$\mathbf{H}_d = -N \mathbf{I} / \mu_0$$

N : Demagnetization coefficient, depend on the shape

Magnetostatic energy:

$$E_d = -\frac{1}{2} \mathbf{I} \mathbf{H}_d = \frac{1}{2\mu_0} N_d I^2$$

For anisotropic materials,

$$H_{dx} = -N_x I_x / \mu_0 \quad H_{dy} = -N_y I_y / \mu_0 \quad H_{dz} = -N_z I_z / \mu_0$$

$$N_x + N_y + N_z = 1$$

$$E_d = \frac{1}{2\mu_0} (N_x I_x^2 + N_y I_y^2 + N_z I_z^2)$$

1) Uniaxial materials

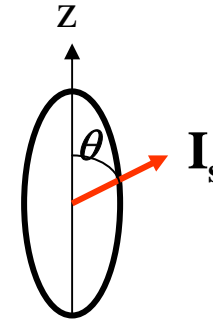
$$N_x = N_y = (1 - N_z)/2$$

$$E_d = \frac{1}{4\mu_0} I_s^2 (3N_z - 1) \cos^2 \theta + \text{const.}$$

$$I_x = I_s \sin \theta \cos \varphi$$

$$I_y = I_s \sin \theta \sin \varphi$$

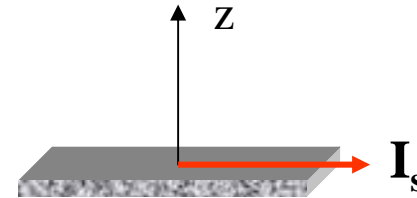
$$I_z = I_s \cos \theta$$



2) Thin films

$$N_x = N_y = 0, \quad N_z = 1$$

Magnetization prefers in-plane.



3) Wires

$$N_z = 0, \quad N_x = N_y = 1/2$$

